CALCULATION OF PARTICLE COLLISIONS IN CONCOMITANT DISPERSED FLOWS

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Relationships are obtained for the relative fraction of the number of particle collisions in concomitant dispersed flows by means of a probability model, and they are verified experimentally.

The intensive application of spraying equipment is explained by the presence in them of a highly extended surface of phase contact. In certain processes, associated with a dispersed phase (mass-exchange processes, chemical reactions, etc.), the principal problems are problems of the interaction of particles in the flows, which determine the quality of the product obtained.

A number of papers [1-3] have appeared recently on the determination of the collision frequency of a certain fixed particle. The most complete data on the interaction of particles of a semidispersed material in flows is given in [4]. At the same time, the results of the investigations carried out by the authors of [4] relate to a large degree to the interaction of particles of a solid semidispersed material and bear a mainly qualitative nature. In this present paper, the problem is to determine the number of particle collisions in concomitant dispersed flows by means of a probability model, which may be systems of liquid — liquid, liquid — friable material, friable material — friable material.

The design scheme is shown in Fig. 1. A cylindrical stream (stream No. 1), for example, of free-falling solid particles, and a conical jet (stream No. 2), formed by spraying a liquid from the atomizer B situated in space coaxially, on intersecting form a zone of possible particle collisions. This zone is bounded by the surfaces of the cone, the cylinder, and the base of the cylinder.

It is well known [6, 8], that the probability of a free (collisionless) range of distance x for particles with diameter d_1, moving in a medium of constant concentration n with particle diameter d_2, is expressed by the following formula, generalized in the case of unequal diameters:

$$P = \exp\left[-\frac{\pi}{4}(d_1+d_2)^2 nx\right]. \tag{1}$$

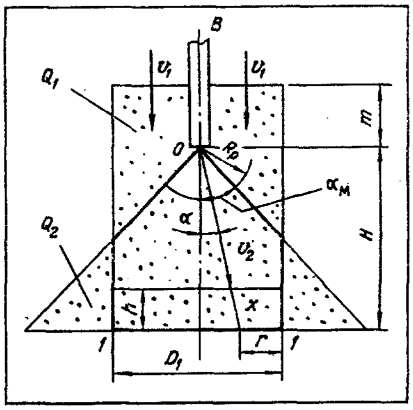

Fig. 1. The problem of particle collisions in concomitant flows.

We obtain the probability of a mean free path of distance x for particles having a velocity $\bar{v}_2$ in a flow with a variable concentration n and variable velocity $\bar{v}_1$ from Eq. (1) by using the theorem of multiplication of probabilities [5] and a limiting process in the exponent of Eq. (1):

$$P = \exp\left(-\frac{\pi}{4}(d_1+d_2)^2\int_0^x \frac{n\sqrt{v_1^2+v_2^2-2v_1v_2\cos\alpha}}{v_2}dx\right), \tag{2}$$

where α is the angle between the vectors of the velocities $\bar{v}_1$ and $\bar{v}_2$.

1. Let us consider the motion of particles of stream No. 2 in stream No. 1 up to its cross section 1 — 1. The velocity $\bar{v}_1$ of

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particles of stream No. 1 at a height h from the cross section 1 — 1 is

$$v_1 = \sqrt{2g(H-h+m)}\,. \tag{3}$$

The concentration of particles in the cylindrical flow is

$$n = \frac{24Q_1}{\pi^2 \sqrt{2g}\sqrt{H-h+m}\,d_1^3 D_1^2}. \tag{4}$$

As shown by preliminary experiments on the spraying of liquids with pneumatic atomizers, the distance r of impact of particles of stream No. 2 on the plane of the cross section of the jet, measured from the axial line of flow, can be assumed with a sufficient degree of accuracy to be distributed according to a semi-normal law [7]:

$$f(r) = \begin{cases} 0, & r < 0, \\ \dfrac{1{,}003\sqrt{2}}{\sqrt{\pi}\,\sigma}\exp\left(-\dfrac{r^2}{2\sigma^2}\right), & 0 < r < \dfrac{D}{2}, \\ 0, & r > \dfrac{D}{2}. \end{cases} \tag{5}$$

Expressing $\sigma = D/6$ using the three-sigma rule, we obtain the following for the flow of liquid particles through an elementary annulus (r, r + dr):

$$dN = \frac{6{,}018\sqrt{2}\,N}{\sqrt{\pi}\,D}\exp\left(-\frac{18r^2}{D^2}\right)dr. \tag{6}$$

The flow per unit time of noncolliding liquid particles through the elementary annular section, according to the law of large numbers [5], is

$$dN_1 = PdN. \tag{7}$$

The total quantity of liquid particles, passing without collision through the section 1 — 1 is obtained by integration of Eq. (7) taking account of Eqs. (2)-(6):

$$N_1 = \frac{18{,}054\sqrt{2\pi}Q_2}{\pi^2 d_2^3 H \operatorname{tg}\alpha_{\text{м}}}\int\limits_0^{\frac{D_1}{2}}\exp\left[-\frac{4{,}5r^2}{H^2\operatorname{tg}^2\alpha_{\text{м}}} - \frac{6Q_1(d_1+d_2)^2}{\pi\sqrt{2g}HD_1^2 d_1^3 v_2} I_1\right]dr,$$

where (8)

$$I_1 = \int\limits_0^{H-\frac{R_p H}{\sqrt{r^2+H^2}}}\sqrt{[v_2^2+2g(H-h+m)]\frac{r^2+H^2}{H-h+m} - 2v_2\sqrt{2g}\frac{H\sqrt{r^2+H^2}}{\sqrt{H-h+m}}}\,dh.$$

2. We consider the motion of a liquid particle up to the lateral surface of the cylinder. The concentration of liquid particles (stream No. 2) in an elementary volume at a distance h from the cross section 1 — 1 and a distance r from the axial line of flow is

$$n = \frac{dN}{v_2 dS\cos\alpha} = \frac{1{,}003\sqrt{2}N\sqrt{(H-h)^2+r^2}\exp\left(-\frac{r^2}{2\sigma_n^2}\right)dr}{2\pi\sqrt{\pi}\,\sigma_h v_2 (H-h)\,rdr}$$

or, taking into account that

$$D = 2H\operatorname{tg}\alpha_{\text{м}},\quad \sigma_h = \frac{D}{6}\cdot\frac{H-h}{H} \text{ и } N = \frac{6Q_2}{\pi d_2^3},$$

we obtain, after transformations,

$$n = \frac{9{,}028\sqrt{2\pi}\,Q_2\sqrt{(H-h)^2+r^2}\exp\left[-\frac{4{,}5r^2}{(H-h)^2\operatorname{tg}^2\alpha_{\text{м}}}\right]}{\pi^3 d_2^3 v_2 r(H-h)^2\operatorname{tg}\alpha_{\text{м}}}. \tag{9}$$

The flow of liquid particles through an elementary lateral surface of the cylinder is

$$dN = nv_2 dS\sin\alpha = \frac{18{,}056\sqrt{2\pi}Q_2 r\exp\left[-\frac{4{,}5r^2}{\operatorname{tg}^2\alpha_{\text{м}}(H-h)^2}\right]}{\pi^2 d_2^3 (H-h)^2\operatorname{tg}\alpha_{\text{м}}}dh. \tag{10}$$

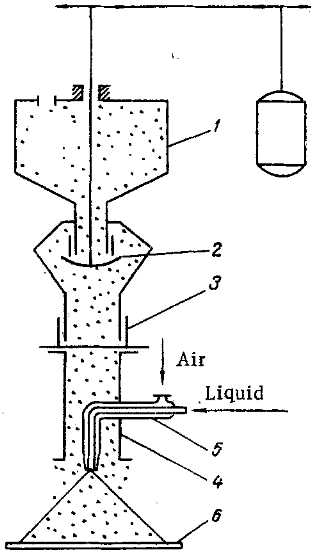

Fig. 2

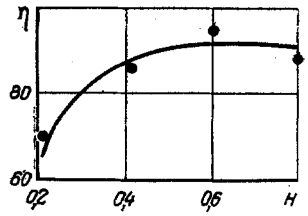

Fig. 3

Fig. 2. Diagram of experimental apparatus for verifying calculations of particle collisions.

Fig. 3. Dependence of the relative fraction of colliding particles (%) on the height of the collision zone (m): the dots represent experimental data.

We obtain the flows of liquid particles passing without collisions through the lateral surface of the zone of interaction, in accordance with Eq. (7) and taking account of Eqs. (2)-(4) and (10):

$$N_2 = \frac{9{,}027\sqrt{2\pi}Q_2D_1}{\pi^2 d_2^3 \operatorname{tg}\alpha_M}\int_0^a \frac{1}{(H-h)^2}\exp\left[-\frac{1{,}12D_1^2}{(H-h)^2\operatorname{tg}^2\alpha_M}-\frac{3Q_1(d_1+d_2)^2}{\pi\sqrt{2g}D_1^2d_1^3v_2}I_2\right]dh, \tag{11}$$

where $a = H - \frac{D_1}{2\operatorname{tg}\alpha_M}$;

$$I_2 = \int_h^b \left([v_2^2 + 2g(H-h+m)]\frac{D_1^2+4(H-h)^2}{H-h+m} - 4v_2\sqrt{2g}(H-h)\times\right.$$

$$\left.\times\sqrt{\frac{D_1^2+4(H-h)^2}{H-h+m}}\right)^{1/2}(H-h)^{-1}dh, \tag{12}$$

and where

$$b = H - \frac{2R_p(H-h)}{\sqrt{D_1^2+4(H-h)^2}}.$$

Let us consider the motion of a stream of solid particles through a conical jet of a liquid component. Taking account of the uniform distribution of the particles of stream No. 1 along the radius, we determine the flow through an elementary cross section (r, r + dr):

$$dN = \frac{48\,Q_1 r\,dr}{\pi D_1^2 d_1^3}. \tag{13}$$

Similar considerations, using formulas (2), (3), (7), and (13), give the following relations for the total flow of noncolliding solid particles passing through the cross section 1 — 1:

when $r = 0$ to $R_p \sin\alpha_M$

$$N_3 = \frac{48\,Q_1}{\pi D_1^2 d_1^3}\int_0^{R_p\sin\alpha_M} r\exp\left[-\frac{2{,}257\sqrt{2\pi}\,Q_2(d_1+d_2)^2}{\pi^2 d_2^3 v_2 r\sqrt{2g}\operatorname{tg}\alpha_M}\int_0^{H-\sqrt{R_p^2-r^2}}\varphi(r,h)\,dh\right]dr, \tag{14}$$

where

$$\varphi(r,\ h)=\left\{[v_2^2+2g(H-h+m)]\frac{r^2+(H-h)^2}{H-h+m}-2v_2\sqrt{2g}(H-h)\times\right.$$

$$\left.\times\sqrt{\frac{r^2+(H-h)^2}{H-h+m}}\right\}^{1/2}\exp\left[-\frac{4{,}5r^2}{(H-h)^2\mathrm{tg}^2\alpha_M}\right](H-h)^{-2};$$

and when $r = R_p \sin\alpha_M$ to $D_1/2$

$$N_4=\frac{48Q_1}{\pi D_1^2d_1^3}\int\limits_{R_p\sin\alpha_M}^{\frac{D_1}{2}}r\exp\left[-\frac{2{,}257\sqrt{2\pi}Q_2(d_1+d_2)^2}{\pi^2d_2^3v_2r\sqrt{2g}\,\mathrm{tg}\,\alpha_M}\int\limits_0^{H-\frac{r}{\mathrm{tg}\alpha_M}}\varphi(r,\ h)\,dh\right]dr. \tag{15}$$

The relative fraction of colliding particles is

$$\eta=\left(1-\frac{N_1+N_2+N_3+N_4}{N}\right)100\%. \tag{16}$$

Calculations of the particle collisions by formulas (8), (11), (12), and (14)-(16) were carried out on the "Odra" computer.

The calculations were verified on an experimental apparatus, which is shown schematically in Fig. 2. The friable component, for which potassium ferrocyanide was used, was admitted from the hopper 1 with the feed plate 2 into the glass cylinder 4 through the distributing grid 3. An aqueous solution of ferric chloride was atomized in the atomizer 5. When the liquid and solid particles interacted, an almost instantaneous dark coloration occurred. The quantity of colliding particles was determined by counting them when trapped on glass [6].

Figure 3 presents graphically some results of the theoretical and experimental determination of the relative fraction of colliding particles, confirming the satisfactory agreement between the experimental data and the data calculated on the "Odra" computer. It can be seen from Fig. 3 that at the start of the region of combined concurrent motion of the streams, the number of particle collisions increases rapidly. With an increase of distance, the increase of the relative fraction of collisions is slowed down, which is explained by the reduction of the flow densities and by the increase of the mean free path of the particles, and for the example given, starting with H = 0.8 m, is almost unchanged. The curve in Fig. 3 was obtained with the following experimental conditions: $Q_1 = 12\cdot10^{-7}$ m³/sec; $Q_2 = 12\cdot10^{-7}$ m³/sec; $d_1 = 0.75\cdot10^{-3}$ m; $d_2 = 0.234\cdot10^{-3}$ m; $D_1 = 0.03$ m; $R_p = 0.03$ m; $\alpha_M = \pi/12$; $v_2 = 0.98$ m/sec.

NOTATION

d_1, d_2	are the particle diameters, m;
v_1, v_2	are the particle velocities, m/sec;
Q_1, Q_2	are the flow rates, m³/sec;
σ_h	is the parameter of the law of distribution at distance h, m;
R_p	is the radius of undecomposed part of jet of liquid, m;
D	is the diameter of cross section of liquid jet at distance H, m;
D_1	is the diameter of column of free-flowing component, m;
η	is the relative fraction of colliding particles, %;
α_M	is the maximum angle of flight of particles from the atomizer;
N	is the total flow of solid and liquid particles per sec.

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